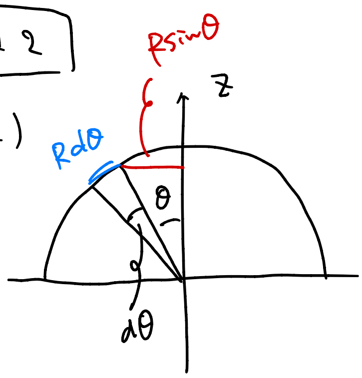
問題2

(1)



左図より　$2\pi R\sin\theta \times R\,d\theta = 2\pi R^2\sin\theta\,d\theta$

∴　$dS = 2\pi R^2\sin\theta\,d\theta$

(2)　電流は単位時間あたりの電荷量であるから、環状電流 dI は、

$$dI = \sigma\,dS\,\omega = 2\pi R^2\sigma\omega\sin\theta\,d\theta$$

(3)　定義より

$$\begin{aligned} dm &= dI \times (R\sin\theta)^2\pi \\ &= 2\pi R^2\sigma\omega\sin\theta\,d\theta \times R^2\sin^2\theta\,\pi \\ &= 2\pi^2R^4\sin^3\theta\,\sigma\omega\,d\theta \end{aligned}$$

$\sin(\underbrace{\theta+2\theta}_{3\theta}) + \sin(\theta)$

$= \sin\theta\cos2\theta + \sin2\theta\cos\theta$

(4)

$$\begin{aligned} m &= \int_0^\pi 2\pi^2R^4\sin^3\theta\,\sigma\omega\,d\theta \\ &= 2\pi^2R^4\sigma\omega\int_0^\pi \sin^3\theta\,d\theta \end{aligned}$$

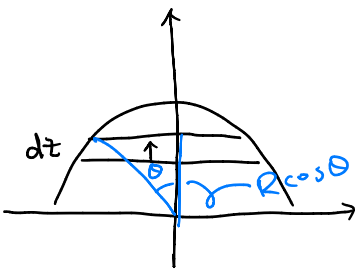
$$\begin{aligned} &= \int_0^\pi \sin^2\theta\cdot\sin\theta\,d\theta \\ &= \int_0^\pi \frac{1-\cos2\theta}{2}\cdot\sin\theta\,d\theta \\ &= \underbrace{\int_0^\pi \frac{1}{2}\sin\theta\,d\theta}_{=1} - \underbrace{\int_0^\pi \frac{1}{2}\sin\theta\cos2\theta\,d\theta}_{=-\frac{1}{3}} = \frac{4}{3} \end{aligned}$$

$\int_0^\pi \sin\theta\,d\theta = \left[-\cos\theta\right]_0^\pi = 2$

$$\begin{aligned} &\int_0^\pi \sin\theta\cos2\theta\,d\theta \\ &= \frac{1}{2}\int_0^\pi(\sin3\theta - \sin\theta)\,d\theta \\ &= -\left[\frac{1}{6}\cos3\theta - \frac{1}{2}\cos\theta\right]_0^\pi \\ &= \frac{1}{3} - 1 = -\frac{2}{3} \end{aligned}$$

∴　$m = \frac{8}{3}\pi^2R^4\sigma\omega$

(5)　ウ



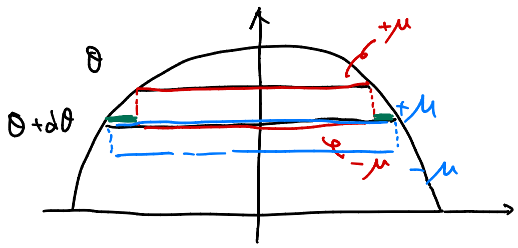
(6)　$dI = n\,dz$

$z = R\cos\theta$

$|dz| = R\sin\theta\,d\theta$ より　$dI = nR\sin\theta\,d\theta$

$dI = 2\pi R^2 \sigma\omega \sin\theta \, d\theta$ と比較して $\eta = 2\pi R \sigma \omega$

(7)



円柱表面と球面ズレは、

$$\pi R^2 \left(\sin^2(\theta + d\theta) - \sin^2\theta \right)$$

$$= \pi R^2 \, 2 \sin\theta \cos\theta \, d\theta$$

ゆえに、球面上に $2\eta \pi R^2 \sin\theta \cos\theta \, d\theta$ の磁荷が存在する。

球面上の面密度は、$\dfrac{2\eta \pi R^2 \sin\theta \cos\theta \, d\theta}{\pi R^2 \sin^2\theta}$

$$= 2\eta \frac{\cos\theta}{\sin\theta} d\theta$$